Phenomenology of Soft Terms in the Presence of Nonvanishing

Hidden Sector Potential Energy

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Abstract

We argue that the hidden sector potential energy is generically of order the intermediate scale although the true cosmological constant does vanish. This would significantly change the predicted values of soft parameters for a variety of supergravity models including those derived from string theory. We stress that this point is particularly relevant for supergravity models in which the soft masses of observable fields vanish at tree level. Implications of a nonzero hidden sector potential energy for low energy phenomenology are also studied.

1. Introduction

   With the accumulating precision data, and the prospect for the operation of LEPII and upgraded Tevatron in the near future, the phenomenological studies of models beyond the standard model have become one of the major issues in particle physics. The front runner among these models is the supergravity models (SUGRA) [1]. In SUGRA, the low energy effective theory is described by supersymmetric terms plus soft supersymmetry breaking terms of order $m_{3/2}$ which is of order the electroweak scale. For the electroweak scale phenomenology in the minimal SUGRA, there are five important parameters, the common scalar mass (m_0), the gaugino mass $(m_{1/2})$, $\tan\beta = v_2/v_1$, A, and the top quark mass m_t. μ and B are determined by the minimization condition. With the anticipated confirmation of the top quark mass at $m_t = 174 \pm 17$ GeV, there will be four parameters in SUGRA phenomenology. In addition, if supersymmetry is broken by the F-component of the dilaton field in string theory, there would exist two more relations defined *at the unification scale*[1], $m_{1/2} = \sqrt{3}m_0$ and $A/\lambda = -\sqrt{3}m_0$, where λ denotes generic Yukawa coupling constant [2,3], while one has a different boundary condition for a different route of supersymmetry breaking, for example in no scale models one would have $m_0 = 0$ and $A = 0$ at tree level [4]. Thus it is of utmost interest to figure out what are the boundary values of soft parameters to get an idea of the origin of the supersymmetry breaking.

   As is well known, the soft scalar mass m_0 and the B coefficient at the unification scale depend on the value of the hidden sector potential energy V_h [5,6]. When one computes these soft parameters for a given SUGRA model, particularly when one derives the above mentioned relations between soft parameters, it has been assumed that V_h does vanish [1]. This would be a consistent choice for the evaluation of soft parameters *at tree level* since the tree level value of the hidden sector potential energy corresponds to the tree level

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[1]Here the unification scale does not necessarily mean the scale where the gauge coupling constants are unified. It can be either the Planck scale or the string scale.

cosmological constant.

However, in order to obtain a physically meaningful result one needs to include quantum effects. In the usual procedure of taking quantum effects into account, one considers renormalization group equations in which logarithmically divergent corrections can be properly included. The boundary values of the renormalized soft parameters at the unification scale have been still chosen to the values obtained by assuming the hidden sector potential energy to be zero. However, it has been recently pointed out [5] that if one includes quantum corrections then the hidden sector potential energy V_h can be sizable, large enough to significantly change the predicted values of soft parameters, while the true cosmological constant V_{eff} still does vanish. The essential point was that, although they are the same at tree level, V_{eff} and V_h receive quantum corrections δV_{eff} and δV_h which are *completely independent* from each other. In Ref. [5], an argument for a sizable V_h was given for models with a large number of observable matter multiplets, $N = O(8\pi^2)$, whose soft scalar masses are comparable to the gravitino mass $m_{3/2}$. In such models, δV_{eff} would be dominated by the quadratically divergent one loop contribution from the matter multiplets, and then $\delta V_{\text{eff}} \simeq N m_0^2 \Lambda^2 / 16\pi^2$ where m_0 denotes the common soft scalar mass and Λ is the cutoff scale of the model[2]. The hidden sector potential energy V_h also receives one loop correction, but δV_h is expected to be of order $m_{3/2}^2 \Lambda^2 / 16\pi^2$. Since we anticipate that our SUGRA model is replaced by a well-behaved theory (for instance string theory) around $\kappa^{-1} \equiv M_{Pl}/\sqrt{8\pi}$, the cutoff Λ is expected to be of order κ^{-1}. Then for $N = O(8\pi^2)$ and m_0 comparable to $m_{3/2}$, the vanishing true cosmological constant, viz $V_0 + \delta V_{\text{eff}} = 0$ where V_0 is the constant vacuum energy density at tree level, implies that the magnitude of $\kappa^2 V_h$ can be comparable to m_0^2. This means that it is not a good approximation to ignore the V_h-dependent part when one computes soft parameters for a given SUGRA model. Note that here a nonzero $\kappa^2 V_h$ is

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[2]In Ref. [5], the whole results were given in terms of the three momentum cutoff Λ_3. Here we will use the more familiar four momentum cutoff Λ_4. Note that $\Lambda_4^2 = 2\Lambda_3^2$.

essentially due to quantum corrections, but it can be sizable because of the large value of N. In this regard, one may recall that the conventional logarithmic quantum corrections are important because of the large value of $\ln(\Lambda^2/m_0^2)$.

The point that the V_h-dependent part of m_0^2 and B can be sizable is particularly relevant for models in which the soft scalar masses and the gaugino masses at the unification scale *vanish at tree level*. In fact, such a case occurs quite often in string-inspired supergravity models [6]. In this case, nonzero soft masses arise usually due to string loop threshold effects. The resulting soft masses are then suppressed by $g^2/16\pi^2$ compared to the gravitino mass or to the typical mass of hidden sector fields, e.g. moduli fields, triggerring spontaneous supersymmetry breaking. Then both the nonzero soft masses and the nonzero $(V_h - V_{\text{eff}})$ are the consequence of quantum corrections at the same order, which means that the V_h-dependent part of m_0^2 and B is essentially the same order as the other parts. Clearly then V_h-dependent part must be taken into account for a consistent calculation of soft parameters. Furthermore, as we will see later, the same argument holds true even for models in which one loop quadratically divergent vacuum energy density does vanish, whose motivation has been recently discussed in Ref. [7,8].

With the observation made above, one can introduce an additional parameter ϵ which shows the importance of the V_h-dependent part of soft parameters:

$$\kappa^2 V_h \equiv -\epsilon m_0^2. \tag{1}$$

It becomes then necessary to know the value of ϵ in order to get an idea of the boundary values of the renormalized soft parameters. In this paper, we wish to elaborate the arguments of Ref. [5] and study how sensitive are the low energy physics at the electroweak scale to the presumably unknown parameter ϵ defined in Eq. (1), mainly focusing on string inspired supergravity models.

2. Hidden Sector Potential Energy and Expressions for Soft Parameters

In this section, we elaborate the arguments of Ref. [5] in the context of a SUGRA model

with the following Kähler potential and superpotential[3] expanded in powers of the observable fields ϕ^i and $\bar{\phi}^{\bar{i}}$:

$$K = \tilde{K}(h, \bar{h}) + Z_{i\bar{j}}(h, \bar{h})\phi^i \bar{\phi}^{\bar{j}} + \frac{1}{2}(Y_{ij}(h, \bar{h})\phi^i \phi^j + \text{h.c.}) + ...,$$

$$W = \tilde{W}(h) + \frac{1}{2}\tilde{\mu}_{ij}(h)\phi^i \phi^j + \frac{1}{3}\tilde{\lambda}_{ijk}(h)\phi^i \phi^j \phi^k + ... \tag{2}$$

Here the hidden sector fields triggering SUSY breaking are collectively denoted by h^α, and the observable sector fields ϕ^i include the quarks, leptons, and the two Higgs doublets H_1 and H_2.

To obtain the effective action S_ϕ of the observable fields ϕ^i, we integrate out the hidden sector fields

$$\exp(iS_\phi) = \int [\mathcal{D}h] \exp(iS), \tag{3}$$

where S is the full supergravity action, and $[\mathcal{D}h]$ includes the integration of the gravity multiplet $(g_{\mu\nu}, \psi_\mu)$ over a background spacetime metric $\bar{g}_{\mu\nu}$ with macroscopic wavelength. Since the high momentum modes of ϕ^i are not integrated out yet, S_ϕ is defined at the cutoff scale Λ in the sense of Wilson. Thus to study the low energy physics of ϕ^i, one still needs to scale the renormalization point down to the weak scale. In the flat limit, S_ϕ would be characterized by the effective superpotential of global SUSY [1]

$$W_{\text{eff}} = \frac{1}{3}\lambda_{ijk}\phi^i \phi^j \phi^k + \frac{1}{2}\mu_{ij}\phi^i \phi^j, \tag{4}$$

and also the soft breaking part of the form

$$\mathcal{L}_{\text{soft}} = m_{i\bar{j}}^2 \phi^i \bar{\phi}^{\bar{j}} + (\frac{1}{3}A_{ijk}\phi^i \phi^j \phi^k + \frac{1}{2}B_{ij}\phi^i \phi^j + \text{h.c.}). \tag{5}$$

Supersymmetry breaking may have the seed in the hidden sector gaugino condensation [9]. In this case, we interpret our SUGRA model as the one obtained after integrating out

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[3]In the minimal supersymmetric standard model, the only gauge and R-parity invariant term of the type $\phi^i \phi^j$ is $\mu H_1 H_2$.

the hidden gauge multiplets (and also the hidden gauge non-singlet matter multiplets if they exist). Note that the hidden gauge multiplets can be integrated out without breaking supersymmetry, and as a result the effects of integration can be summarized by the superpotential $\tilde{W}$ of the gauge singlet hidden fields h^α whose nonzero F-components are responsible for supersymmetry breaking[4].

Among the coefficients in W_{eff} and $\mathcal{L}_{\text{soft}}$, those which depend on the hidden sector scalar potential V_h are relevant for us. On dimensional ground, we expect a correction of order $\kappa^2 V_h$ for $m_{i\bar{j}}^2$ and B_{ij} while a correction of order $\kappa^3 V_h \ll m_{3/2}$ for A_{ijk} and the gaugino masses. (Here $\kappa = \sqrt{8\pi}/M_{Pl}$.) Thus, for phenomenology, the important correction can arise only in the soft scalar masses and the B terms.

To calculate soft terms in the scalar potential, we first expand the supergravity action in powers of the observable fields ϕ^i. Integrating out the hidden fields (see Eq. (3)), the following soft scalar masses and the B terms (for un-normalized fields) are obtained [2]:

$$m_{i\bar{j}}^2 \phi^i \bar{\phi}^{\bar{j}} + \frac{1}{2}(B_{ij}\phi^i\phi^j + \text{h.c.}), \tag{6}$$

where[5]

$$m_{i\bar{j}}^2 = \langle (\kappa^2 V_h + m_{3/2}^2)Z_{i\bar{j}} - F^\alpha \bar{F}^{\bar{\beta}} R_{\alpha\bar{\beta}i\bar{j}} \rangle$$

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[4]Recently, there has been found a discrepancy [10] between effective field theory calculation [11] and the direct calculation [10] of soft parameters in the gaugino condensation model of supersymmetry breaking. However, as pointed out in Ref. [10], we believe that the two calculations should agree if one includes the neglected terms of order $1/M_{Pl}^2$ in the effective field theory framework. In this case, supersymmetry breaking by gaugino condensation can be also described by the above superpotential given in Eq. (2).

[5]In fact, for a consistent study of quantum effects, one needs to include in the expressions below the part depending upon the hidden sector fermions. Here we will ignore this point since our main point is _unchanged._

$$= \langle \frac{2}{3}\kappa^2 V_h Z_{i\bar{j}} + F^\alpha \bar{F}^{\bar{\beta}}(\frac{1}{3}K_{\alpha\bar{\beta}}Z_{i\bar{j}} - R_{\alpha\bar{\beta}i\bar{j}})\rangle,$$

$$B_{ij}= \langle(\kappa^2 V_h + 2m_{3/2}^2)Y_{ij} + m_{3/2}(F^\alpha D_\alpha Y_{ij} - \bar{F}^{\bar{\alpha}}D_{\bar{\alpha}}Y_{ij}) - F^\alpha \bar{F}^{\bar{\beta}}D_\alpha D_{\bar{\beta}}Y_{ij}\rangle$$

$$= \langle \frac{1}{3}\kappa^2 V_h Y_{ij} + m_{3/2}(F^\alpha D_\alpha Y_{ij} - \bar{F}^{\bar{\alpha}}D_{\bar{\alpha}}Y_{ij}) + F^\alpha \bar{F}^{\bar{\beta}}(\frac{2}{3}K_{\alpha\bar{\beta}} - D_\alpha D_{\bar{\beta}})Y_{ij}\rangle. \tag{7}$$

Here we set $\tilde{\mu}_{ij}$ to zero since the associated B-coefficients are not significantly affected by the hidden sector potential energy, and

$$\bar{F}^{\bar{\alpha}} = e^{-i\theta}e^{\tilde{K}/2}\tilde{K}^{\beta\bar{\alpha}}(\partial_\beta \tilde{W} + \tilde{W}\partial_\beta \tilde{K}), \quad \theta = \arg(\tilde{W}),$$

$$D_\alpha Y_{ij} = \partial_\alpha Y_{ij} - \Gamma^l_{\alpha(i}Y_{j)l}, \quad D_{\bar{\alpha}}Y_{ij} = \partial_{\bar{\alpha}}Y_{ij},$$

$$R_{\alpha\bar{\beta}i\bar{j}} = \partial_\alpha\partial_{\bar{\beta}}Z_{i\bar{j}} - \Gamma^l_{\alpha i}\Gamma^{\bar{m}}_{\bar{\beta}\bar{j}}Z_{l\bar{m}}, \quad \Gamma^l_{\alpha i} = Z^{l\bar{m}}\partial_\alpha Z_{i\bar{m}}, \tag{8}$$

and the gravitino mass $m_{3/2}$ and the hidden sector scalar potential V_h are given by

$$m_{3/2} = e^{\tilde{K}/2}|\tilde{W}|,$$

$$V_h = F^\alpha \bar{F}^{\bar{\beta}}\tilde{K}_{\alpha\bar{\beta}} - 3m_{3/2}^2. \tag{9}$$

The bracket means the average over the hidden sector fields. For example,

$$\langle V_h\rangle = \int [\mathcal{D}h]V_h(h^\alpha)\exp(iS_h)/\int [\mathcal{D}h]\exp(iS_h), \tag{10}$$

where S_h is the supergravity action of the hidden sector fields alone.

Eq. (7) shows that $m_{i\bar{j}}^2$ and B_{ij} associated with the Kähler potential term $Y_{ij}\phi^i\phi^j$ [12] depend on $\langle V_h Z_{i\bar{j}}\rangle$ and $\langle V_h Y_{ij}\rangle$ respectively [5,6]. In the following, we assume the factorization approximation is valid and thus

$$\langle V_h Z_{i\bar{j}}\rangle \simeq \langle V_h\rangle\langle Z_{i\bar{j}}\rangle, \quad \langle V_h Y_{ij}\rangle \simeq \langle V_h\rangle\langle Y_{ij}\rangle. \tag{11}$$

For local SUSY broken by a nonzero value of the auxiliary component F^α, *unless* one implements a fine tuning of some parameters in the hidden sector superpotential $\tilde{W}$, the typical size of $\langle V_h\rangle$ would be of $O(|F^\alpha|^2) = O(\kappa^{-2}m_{3/2}^2)$. In most of the previous studies, motivated by the vanishing cosmological constant, the expectation value of the hidden sector scalar potential $\langle V_h\rangle$ was simply assumed to be zero. However, as we argue below, $\kappa^2\langle V_h\rangle$ is

generically of order the soft scalar mass squared of observable fields, and thus ignoring the $\langle V_h \rangle$-dependent part of soft parameters is *not* a sensible approximation.

In this regard, a simple but essential point is that $\langle V_h \rangle$ is not the true cosmological constant. The fully renormalized cosmological constant V_{eff} at low energy is obtained by integrating out all the fields in the theory,[6]

$$\exp(i \int d^4 x \sqrt{\bar{g}} V_{\text{eff}}) = \int [\mathcal{D}\phi \mathcal{D}h] \exp(i \int d^4 x \sqrt{g} \mathcal{L}) \tag{12}$$

where $[\mathcal{D}\phi]$ represents the integration over all the observable gauge and matter multiplets. In the classical approximation, both $\langle V_h \rangle$ of Eq. (10) and V_{eff} of Eq. (12) are simply the classical potential in S_h. Therefore, we have

$$(V_{\text{eff}})_{\text{tree}} = \langle V_h \rangle_{\text{tree}}. \tag{13}$$

However, beyond the tree level, obviously $\langle V_h \rangle$ is *not* the same as the fully renormalized cosmological constant. For instance, $\langle V_h \rangle$ of Eq. (10) is completely independent of the observable sector dynamics, while V_{eff} of Eq. (12) depends on.

To proceed, let us consider a toy model which is calculable and at the same time shows the point essential to us. The model contains a single hidden chiral field h and N observable chiral fields ϕ^i with the following lagrangian:

$$\mathcal{L}_{\text{toy}} = \partial_\mu h \partial^\mu h^* + \partial_\mu \phi^i \partial^\mu \phi^{i*} + \frac{1}{2} \bar{\psi}(i\gamma^\mu \partial_\mu - m_F)\psi + \frac{1}{2} \bar{\chi}^i i\gamma^\mu \partial_\mu \chi^i$$
$$- [V_0 + m_B^2 h h^* + \{\kappa^2(V_0 + m_B^2 h h^*) + \bar{m}_0^2\}\phi^i \phi^{i*}], \tag{14}$$

where ψ and χ^i denote the superpartner Majorana fermions of h and ϕ^i, respectively. Here the theory is regulated by an explicit cut-off scale Λ which is of order $\kappa^{-1} = M_{Pl}/\sqrt{8\pi}$. The mass parameters m_B, m_F and $\bar{m}_0$ are of order the electroweak scale, and the constant vacuum energy density V_0 can be of $O(\kappa^{-2}\bar{m}_0^2)$. In this toy model, supersymmetry appears

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[6]The long wave length metric $\bar{g}_{\mu\nu}$ is treated as background and is not integrated out to see the gravitational effect at low energy.

to be explicitly broken for nonzero values of the parameters $\kappa^2 V_0$, $(m_B^2 - m_F^2)$, $\bar{m}_0^2$, and m_B^2 which are introduced to mimic the potentially complicated hidden sector dynamics which would trigger spontaneous supersymmetry breaking. Thus by definition these supersymmetry breaking parameters can *not* be significantly larger than $m_{3/2}^2$.

The hidden sector potential energy of our toy model is

$$V_h = V_0 + m_B^2 h h^*, \tag{15}$$

and the soft mass of the observable ϕ^i is

$$m_0^2 = \bar{m}_0^2 + \kappa^2 \langle V_h \rangle. \tag{16}$$

Obviously at tree level,

$$(V_{\text{eff}})_{\text{tree}} = \langle V_h \rangle_{\text{tree}} = V_0. \tag{17}$$

Using Eqs. (10) and (12), one can easily compute the cosmological constant V_{eff} and the expectation value of the hidden sector potential energy $\langle V_h \rangle$ at the full quantum level. The results are

$$\langle V_h \rangle = V_0 + \frac{1}{16\pi^2} m_B^2 \Lambda^2,$$
$$V_{\text{eff}} = V_0 + V_1 + V_2 + O(m_B^4), \tag{18}$$

where V_1 and V_2 denote the one loop and the two loop corrections to the vacuum energy density:

$$V_1 = \frac{1}{16\pi^2} \Lambda^2 [(m_B^2 - m_F^2) + N(\bar{m}_0^2 + \kappa^2 V_0)],$$
$$V_2 = \frac{N}{16\pi^2} \left(\frac{\kappa^2 \Lambda^2}{16\pi^2} \right) m_B^2 \Lambda^2. \tag{19}$$

Note that the above result of $\langle V_h \rangle$ is exact and also that of V_{eff} is exact up to small corrections of $O(m_B^4)$.

The results of Eqs. (18) and (19) show apparently that the quantum correction to $\langle V_h \rangle$ and the correction to V_{eff} are completely independent from each other. Imposing the condition of vanishing cosmological constant, viz $V_{\text{eff}} = 0$, we find

$$\kappa^2 \langle V_h \rangle = -\frac{\kappa^2 \Lambda^2}{16\pi^2} [N(\bar{m}_0^2 + \kappa^2 V_0) + \frac{N\kappa^2 \Lambda^2}{16\pi^2}) m_B^2 - m_F^2],$$
$$= -\frac{\kappa^2 \Lambda^2}{16\pi^2} [Nm_0^2 - m_F^2]$$
$$\equiv -\epsilon m_0^2. \tag{20}$$

Using the above results of our toy model, we can discuss the size of ϵ for a variety of cases. Before going further, let us note that our toy model does not contain gauge multiplets. In realistic models, the quantum corrections to V_{eff} would receive an additional contribution from the observable sector gaugino masses, while $\langle V_h \rangle$ is not affected by the observable sector dynamics. However, in most interesting SUGRA models the gaugino mass $m_{1/2}$ is either comparable to or significantly smaller than m_0. For $m_{1/2}$ comparable to m_0, we will assume that the number of chiral matter multiplets is significantly greater than that of gauge multiplets. Then the following discussion in the context of our toy model will be applied also for realistic models.

Let us first consider the case that all of $\bar{m}_0$, m_B, and m_F are comparable to $m_{3/2}$. (Note that $\bar{m}_0$ and m_0 denote the tree level mass and the quantum corrected mass of observable scalar fields at the unification scale, while m_B and m_F denote the typical mass of the hidden sector scalar and the hidden sector fermion, respectively.) The dilaton dominated scenario in string theory and also generic SUGRA models with the flat Kähler potential would correspond to this case. In this case, Eq. (20) implies $\epsilon = O(N\kappa^2\Lambda^2/16\pi^2)$. Then ϵ can be significant if the cutoff[7] is comparable to κ^{-1} and $N = O(8\pi^2)$. Furthermore, in this case, it is likely that ϵ is positive [5].

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[7] Here the cut off scale Λ would not be exactly the same as the frequently quoted mass scale $M_{\text{GUT}} = e^{(1-\gamma)/2} 3^{-3/4} g\kappa^{-1}/\sqrt{2\pi}$ in string theory [13] though both Λ and M_{GUT} are comparable to the string scale $M_{\text{st}} = g\kappa^{-1}$. Note that a precise definition of M_{GUT} is a matter of convention and the above choice was made for a particular way to incorporate the threshold correction to the renormalized gauge coupling constants [13].

Even more interesting case is the one in which m_0^2 is of order $g^2 m_{3/2}^2/16\pi^2$, while m_B and m_F are still comparable to $m_{3/2}$. This would be the case for many of the moduli dominated supersymmetry breaking scenarios in string theory which give vanishing soft scalar and gaugino masses of observable fields *at tree level*. In such cases, soft masses are induced by string loop threshold corrections [6]. The resulting $m_0/m_{3/2}$ and $m_{1/2}/m_{3/2}$ are of order $\sqrt{g^2/16\pi^2}$ and $g^2/16\pi^2$, respectively, implying that $m_{1/2}$ is significantly smaller than m_0. Then Eq. (20) gives $\epsilon = O(\frac{\kappa^2\Lambda^2}{16\pi^2}\frac{16\pi^2}{g^2})$. This shows that ϵ does *not* represent a correction to the leading result, but rather it corresponds to the ratio between two independent one loop effects and thus of order unity in general.

Recently an interesting class of string inspired no scale SUGRA models have been considered in Ref. [7,8]. The peculiar property of those models was that both the tree level vacuum energy density and the one loop quadratically divergent vacuum energy density vanish, viz $V_0 = V_1 = 0$. Typically such models also have the vanishing soft masses of observable fields at tree level, viz $\bar{m}_0 = 0$, while some of hidden sector fields have masses of order $m_{3/2}$. Then with $\bar{m}_0 = V_0 = V_1 = 0$, we find

$$\langle V_h \rangle = \frac{\kappa^2\Lambda^2}{16\pi^2}(1 - \frac{N\kappa^2\Lambda^2}{16\pi^2})m_B^2 \equiv -\epsilon m_0^2. \tag{21}$$

Again with $m_0^2 = O(g^2 m_{3/2}^2/16\pi^2)$ induced by string loop threshold and $m_B = O(m_{3/2})$, we find ϵ is essentially of order unity in this case also[8].

So far, we have argued that the boundary values of scalar mass m_0^2 and the B parameter in generic SUGRA models can be significantly changed compared to those in the naive approach assuming the hidden sector potential V_h to zero. Thus when one computes soft parameters for a given SUGRA model, one needs to introduce an additional parameter ϵ which is defined

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[8] In the models with $V_0 = V_1 = 0$ which have been considered in Ref. [7], some of hidden scalars are extremely light, having masses of order $\kappa m_{3/2}^2$. However our argument here is valid as long as there exists any hidden scalar with its mass $m_B = O(m_{3/2})$, which is always the case for the models of Ref. [7].

by $\langle V_h \rangle \equiv -\epsilon m_0^2$ to parameterize the importance of the contribution from nonzero hidden sector potential energy. In most of interesting cases, ϵ could be (or was essentially) of order unity. We wish to stress again that for the case that the soft masses of observable sector vanish at tree level, which is the case that occurs quite often in string inspired SUGRA, ϵ does *not* represent a correction to the leading results, but rather corresponds to the ratio between two independent one loop effects. Then theories predicting soft parameters would suffer from an uncertainty associated with the potentially sizable value of ϵ. In the next section, we investigate how sensitive the low energy phenomenology is to the parameter ϵ.

3. The ϵ Dependence of Masses of Superpartners at Low Energy

In this section, we will evolve the ϵ-dependent boundary values to the electroweak scale to see how sensitive the low energy physical parameters are to the value of ϵ. To be explicit, we consider two specific scenarios for supersymmetry breaking which lead to very distinctive predictions of soft parameters: the dilaton-dominated model [2,3] and the moduli-dominated orbifold model with small string loop threshold corrections and all modular weights of matters fields being -1 [6].

For a numerical study, we assume the minimal particle content in the observable sector, viz. the particles of the minimal supersymmetric standard model (MSSM). If there exist more particles, with masses between the unification scale and the electroweak scale, transforming nontrivially under $SU(3) \times SU(2) \times U(1)$, our results would be changed accordingly. For our purpose, it is sufficient to consider the following renormalization group equations [14] keeping only the leading terms in the mass hierarchy in the three generation MSSM.

$$\frac{dM_a}{dt} = \frac{2}{16\pi^2} b_a g_a^2 M_a \,,$$

$$\frac{dA_t}{dt} = \frac{2}{16\pi^2} \left(\sum c_a g_a^2 M_a + 6\lambda_t^2 A_t + \lambda_b^2 A_b \right) \,,$$

$$\frac{dA_b}{dt} = \frac{2}{16\pi^2} \left(\sum c_a' g_a^2 M_a + 6\lambda_b^2 A_b + \lambda_t^2 A_t + \lambda_\tau^2 A_\tau \right) \,,$$

$$\frac{dA_\tau}{dt} = \frac{2}{16\pi^2} \left(\sum c_a'' g_a^2 M_a + 3\lambda_b^2 A_b + 4\lambda_\tau^2 A_\tau \right) \,,$$

$$\frac{dB}{dt} = \frac{2}{16\pi^2} \left(\frac{3}{5} g_1^2 M_1 + 3g_2^2 M_2 + 3\lambda_b^2 A_b + 3\lambda_t^2 A_t + \lambda_\tau^2 A_\tau \right) \,,$$

$$\frac{d\mu}{dt} = \frac{\mu}{16\pi^2}\Big(-\frac{3}{5}g_1^2 - 3g_2^2 + 3\lambda_t^2 + 3\lambda_b^2 + \lambda_\tau^2 \Big) \,,$$

$$\frac{dM_{H_1}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + 3\lambda_b^2 X_b + \lambda_\tau^2 X_\tau \Big) \,,$$

$$\frac{dM_{H_2}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + 3\lambda_t^2 X_t \Big) \,,$$

$$\frac{dM_{Q_L}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{1}{15}g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{16}{3}g_3^2 M_3^2 + \lambda_t^2 X_t + \lambda_b^2 X_b \Big) \,,$$

$$\frac{dM_{t_R}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{16}{15}g_1^2 M_1^2 - \frac{16}{3}g_3^2 M_3^2 + 2\lambda_t^2 X_t \Big) \,,$$

$$\frac{dM_{b_R}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{4}{15}g_1^2 M_1^2 - \frac{16}{3}g_3^2 M_3^2 + 2\lambda_b^2 X_b \Big) \,,$$

$$\frac{dM_{L_L}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \lambda_\tau^2 X_\tau \Big) \,,$$

$$\frac{dM_{\tau_R}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{12}{5}g_1^2 M_1^2 + 2\lambda_\tau^2 X_\tau \Big) \,, \tag{22}$$

and for the two light generations,

$$\frac{dA_u}{dt} = \frac{2}{16\pi^2}\Big(\sum c_a g_a^2 M_a + \lambda_t^2 A_t \Big) \,,$$

$$\frac{dA_d}{dt} = \frac{2}{16\pi^2}\Big(\sum c_a' g_a^2 M_a + \lambda_b^2 A_b + \frac{1}{3}\lambda_\tau^2 A_\tau \Big) \,,$$

$$\frac{dA_e}{dt} = \frac{2}{16\pi^2}\Big(\sum c_a'' g_a^2 M_a + \lambda_b^2 A_b + \frac{1}{3}\lambda_\tau^2 A_\tau \Big) \,,$$

$$\frac{dM_{q_L}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{1}{15}g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{16}{3}g_3^2 M_3^2 \Big) \,,$$

$$\frac{dM_{u_R}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{16}{15}g_1^2 M_1^2 - \frac{16}{3}g_3^2 M_3^2 \Big) \,,$$

$$\frac{dM_{d_R}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{4}{15}g_1^2 M_1^2 - \frac{16}{3}g_3^2 M_3^2 \Big) \,,$$

$$\frac{dM_{l_L}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 \Big) \,,$$

$$\frac{dM_{e_R}^2}{dt} = \frac{2}{16\pi^2}\Big(-\frac{12}{5}g_1^2 M_1^2 \Big) \,, \tag{23}$$

where M_a $(a = 1,2,3)$ are the gaugino masses of $SU(3) \times SU(2) \times U(1)$, M_ϕ are the scalar masses, and

$$b_a = (\frac{33}{5}, 1, -3) \,,$$

$$c_a = (\frac{13}{15}, 3, \frac{16}{3}) \,,$$

$$c_a' = (\frac{7}{15}, 3, \frac{16}{3}) \,,$$

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$$c''_a = (\frac{9}{5}, 3, 0) \,,$$

$$X_t = M^2_{Q_L} + M^2_{t_R} + M^2_{H_2} + A^2_t \,,$$

$$X_b = M^2_{Q_L} + M^2_{b_R} + M^2_{H_1} + A^2_b \,,$$

$$X_\tau = M^2_{L_L} + M^2_{\tau_R} + M^2_{H_1} + A^2_\tau \,. \tag{24}$$

Here, $t = \ln(P/M_U)$ for the renormalization point P and the unification scale M_U, and the factors c_a, c'_a, and c''_a are given by a sum over the fields in the relevant Yukawa coupling, e.g. $c_a = \sum_f c_a(f) = c_a(H_2) + c_a(Q) + c_a(U^c)$. The Yukawa couplings are defined by

$$\lambda_t = \frac{\sqrt{2}m_t}{v \sin\beta}, \quad \lambda_b = \frac{\sqrt{2}m_b}{v \cos\beta}, \quad \lambda_\tau = \frac{\sqrt{2}m_\tau}{v \cos\beta}. \tag{25}$$

Before performing a numerical analysis, let us summarize the relevant formulae of soft parameters which have been derived from string inspired supergravity models. Following Brignole, Ibanez and Munoz [6], we parametrize the soft parameters at the unification scale in the following manner,

$$
\begin{aligned}
m_0^2 &= \kappa^2 V_h + m_{3/2}^2 - \frac{1}{3}\kappa^2|F|^2(1-\delta_0)\cos^2\theta \\
&= \frac{1}{3}\kappa^2\left[2V_h + |F|^2(\sin^2\theta + \delta_0\cos^2\theta)\right], \\
M_a &= (1+\eta_a)\kappa F(\sin\theta + \delta_a\cos\theta), \\
A &= -\kappa F\sin\theta, \tag{26}
\end{aligned}
$$

where $\kappa = \sqrt{8\pi}/M_{Pl}$, the Goldstino angle θ is defined by $\tan\theta = F_S/F_T$, the small parameters δ_0, δ_a, and η_a represent string loop effects, $|F|^2 = F^\alpha \bar{F}^{\bar\beta} K_{\alpha\bar\beta}$ corresponds to the scale of local supersymmetry breaking, and finally the hidden sector potential energy is given by $V_h = |F|^2 - 3\kappa^{-2}m_{3/2}^2$. Note that F, V_h, and θ are determined by nonperturbative dynamics, i.e. highly depend on the hidden sector superpotential $\tilde{W}(h)$ which is presumed to be generated by nonperturbative effects, while δ_0, δ_a, and η_a are perturbative parameters which can be calculated in string perturbation theory. The parametrization given above is valid in fact for a wide class of superstring models, e.g. large size limit of Calabi-Yau manifolds,

generic dilaton dominated cases, and orbifold models with small string threshold corrections and all modular weights of matter fields being -1.

Then the parametrization taking the possibility of significantly large $\kappa^2 V_h = -\epsilon m_0^2$ into account is given by

$$
\begin{aligned}
m_0^2 &= \frac{1}{3}(1 + \frac{2}{3}\epsilon)^{-1}\kappa^2|F|^2(\sin^2\theta + \delta_0\cos^2\theta), \\
M_a &= (1 + \eta_a)\kappa F(\sin\theta + \delta_a\cos\theta), \\
A &= -\kappa F\sin\theta.
\end{aligned}
\tag{27}
$$

(i) Dilaton–dominated case with small string-loop threshold effect: Here we assume $\sin^2\theta \gg \delta_0, \delta_a, \eta_a$, and thus

$$
\begin{aligned}
m_0^2 &\simeq \frac{1}{3}(1 + \frac{2}{3}\epsilon)^{-1}\kappa^2|F|^2\sin^2\theta = \frac{1}{3 + 2\epsilon}m_{1/2}^2, \\
M_a &\simeq \kappa F\sin\theta = m_{1/2}, \\
A &= -\kappa F\sin\theta = -m_{1/2}.
\end{aligned}
\tag{28}
$$

(ii) Moduli–dominated orbifold case with the modular weights of all matters being -1: Here we assume $\delta_{GS} = 10$ and $\sin\theta = 0$, and then

$$
\begin{aligned}
\delta_0 &= \frac{g^2}{48\pi^2}\delta_{GS} = 10^{-2}, \\
\eta_0 &= 0 \ , \quad \eta_2 = 0.06 \ , \quad \eta_3 = 0.18 \ , \\
\delta_3 &= (\delta_{GS} - 3) \times 4.6 \times 10^{-4} = 3.22 \times 10^{-3}, \\
\delta_2 &= (\delta_{GS} + 1) \times 4.6 \times 10^{-4} = 5.06 \times 10^{-3}, \\
\delta_1 &= (\delta_{GS} + \frac{33}{5}) \times 4.6 \times 10^{-4} = 7.64 \times 10^{-3},
\end{aligned}
\tag{29}
$$

which give

$$
\begin{aligned}
m_0^2 &= 3.33 \times 10^{-3}(1 + \frac{2}{3}\epsilon)^{-1}\kappa^2|F|^2, \\
M_3 &= 3.22 \times 10^{-3}\kappa F, \\
M_2 &= 5.36 \times 10^{-3}\kappa F, \\
M_1 &= 9.02 \times 10^{-3}\kappa F, \\
A &= 0.
\end{aligned}
\tag{30}
$$

The soft supersymmetry breaking parameters relevant for $SU(2) \times U(1)$ breaking are m_0^2, m_t, A, B, and the gaugino masses. Also the supersymmetric Higgs mixing parameter μ contributes to the $SU(2) \times U(1)$ breaking. With these parameters, we search for the mimimum of the potential determining $\langle H_1^0 \rangle = v_1$ and $\langle H_2^0 \rangle = v_2$. We then trade μ and B for $\tan\beta = v_2/v_1$ and $v = \sqrt{v_1^2 + v_2^2}$. In the presence of a nonzero $\kappa^2 V_h = -\epsilon m_0^2$, only m_0^2 and B can be significantly affected. In our numerical analysis for the boundary conditions of Eqs. (28) and (30), only the ϵ-dependence of m_0^2 has been implemented, while B is determined by imposing the radiative electroweak symmetry breaking.

Let us first summarize the results of the dilaton-dominated case (i). As was explained in the previous section, in this case ϵ would represent a potentially large quantum correction associated with the large value of N, the number of observable chiral multiplets. In Fig. 1, we present the masses of the third generation squarks and sleptons at the electroweak scale for a particular choice of parameters with $m_{1/2} = 150$ GeV, $m_t = 170$ GeV, and $\tan\beta = 2$. There ϵ is allowed to vary from zero to one. It shows that the effect of a nonzero hidden sector potential energy for the low energy squark masses is negligible in this case. The reason for this can be traced back to the renormalization group equations for squarks in Eqs. (22) and (23) which show that, for the gluino mass bigger than the squark mass, the low energy squark masses are determined mainly by the ϵ-independent $g_3 M_3$, not by the ϵ-dependent boundary value of m_0^2 at the unification scale. Therefore, changing the boundary value of m_0^2 by order unity does not change the low energy squark masses very much. On the other hand, the change of the slepton masses is somewhat noticeable. For example, for $\epsilon = 0.6$ the slepton masses decrease by 8 %. The renormalization group equation for sleptons still contains the contribution from $g_2 M_2$, but its effect is not as dramatic as that of the squark case since it involves the weak gauge coupling constant g_2. This can be more easily understood from the anaytic expressions for squark and slepton masses in the two light generations given by

$$M_{q_L}^2 \simeq m_0^2 + 8m_{1/2}^2 , \quad M_{l_L}^2 \simeq m_0^2 + 0.63m_{1/2}^2 , \tag{31}$$

where the effect of changing m_0 through ϵ in $M_{q_L}^2 (M_{l_L}^2)$ becomes negligible (pronounced) because of a large (small) coefficient of $m_{1/2}^2$. In Fig. 2, we present the first generation squark masses as a function of ϵ. In Fig. 3, we present the Higgs boson masses as a function of ϵ. The Higgs boson masses are insensitive to ϵ. Similarly in Fig. 4, we show chargino (χ_i^+) and neutralino (χ_i^0) masses. In summary, for the dilaton dominated case (i), except for the slepton masses, corrections to low energy soft parameters due to ϵ are not significant at all.

Let us now consider Case (ii) of moduli-dominated scenario. As was stressed, ϵ in this case corresponds to the ratio between two independent one loop effects, and thus is essentially of order unity. Thus compared to the case (i), ϵ can vary over a wider range, but we restrict it up to 1 to compare with Figs. (1)–(4). (However, keep in mind that ϵ can take values greater than 1.) Furthermore, gaugino masses in Case (ii) are significantly smaller than the soft scalar masses. As a result, low energy soft parameters would be more sensitive to the ϵ-dependent boundary value of m_0^2. In Fig. 5, we present the ϵ dependence of the third generation squark and slepton masses. Here the input masses of $m_{1/2} = 100$ GeV, $m_t = 150$ GeV, and $\tan \beta = 2$ are used. $m_{1/2}$ is the gluino mass at the unification scale and the wino and bino masses at the unification scale are given by Eq. (30). As expected, the moduli dominated scenario has the stronger ϵ dependence. For example, the third generation slepton masses decrease by ~ 15 % when one varies ϵ from 0 to 0.6. In Fig. 6, we present the first generation squark masses. Here also, the ϵ dependence of the masses are pronounced. In Figs. 7 and 8, we present the Higgs boson masses and chargino and neutralino masses, respectively.

4. Conclusion

In the previous studies of soft parameters within supergravity models, the hidden sector potential energy has been usually assumed to vanish. The only motivation for this assumption is the vanishing cosmological constant. In this paper we expanded the discussion of Ref. [5] that although they are the same at tree level the vacuum energy density V_{eff} and the hidden sector potential energy V_h receive completely different quantum corrections. As

shown for the simple model in Sec. 2, V_h (the hidden sector vacuum energy) is generally of order $\kappa^{-2}m_0^2 \equiv m_0^2 M_{Pl}^2/8\pi$ for V_{eff} (the true cosmological constant) = 0, where m_0 denotes the soft mass of observable sector scalar fields at the unification scale. This leads to a physically interesting consequence in particle physics since m_0^2 and B depend on $\kappa^2 V_h$. The value of $\kappa^2 V_h$ of order m_0^2 gives rise to a significant change in the predicted values of soft parameters in a variety of supergravity models.

It should be stressed that the above point is particularly relevant for supergravity models in which the soft masses of observable fields vanish at tree level, which is the case occurring quite often in string inspired supergravity models [6]. In such models, both nonzero m_0^2 and nonzero $(V_h - V_{\text{eff}})$ are the consequence of quantum effects at the same order. Then $\epsilon = -\kappa^2 V_h/m_0^2$ corresponds to the ratio of two independent one loop effects, and thus is essentially of order unity. The same observation holds true for the recently discussed no scale models [7,8] in which both the tree level vacuum energy density and the one loop quadratically divergent vacuum energy density vanish.

In order to see the low energy implication of the nonvanishing V_h, we have performed the renormalization group evolution starting from the ϵ-dependent m_0^2 at the unification scale. We have imposed explicitly the radiative electroweak symmetry breaking. In the dilaton dominated supersymmetry breaking scenario, the slepton masses have a sizable ϵ dependence, but the other masses are insensitive to ϵ. In the moduli dominated scenario, the ϵ dependence of soft masses are much more pronounced.

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FIGURES

FIG. 1. The third generation squark [(a) $\tilde{t}$, $\tilde{b}$] and slepton [(b) $\tilde{\nu}$, $\tilde{\tau}$] masses as a function of ϵ in the dilaton dominated supersymmetry breaking scenario. Stop, sbottom and gluino are represented by $\tilde{t}$, $\tilde{b}$, and $\tilde{g}$, respectively. The masses are in units of GeV. Among two mass eigenstates of $\tilde{t}$, $\tilde{b}$ and $\tilde{\tau}$, the heavier ones are suffixed with 2 and the lighter ones with 1. We use the following set of parameters, $m_{\tilde{g}} = 325$ GeV, $m_t = 170$ GeV, and $\tan\beta = 2$. These masses are the values at the electroweak scale. The universal gaugino mass of 150 GeV is given at the unification scale.

FIG. 2. The first generation squark ($\tilde{u}$, $\tilde{d}$) masses as a function of ϵ in the dilaton scenario. The input parameters are the same as in the Fig. 1. We do not present the first generation slepton masses, since they are almost the same as the third generation slepton masses.

FIG. 3. The Higgs boson masses as a function of ϵ in the dilaton scenario. The lightest Higgs boson is h, and the neutral pseudoscalar Higgs boson is A. The input parameters are the same as in the Fig. 1.

FIG. 4. Same as in the Fig. 1 except that the chargino and neutralino masses are presented.

FIG. 5. The third generation squark and slepton masses as a function of ϵ in the moduli dominated supersymmetry breaking scenario. The input parameters are $m_{1/2} = 100$ GeV, $m_t = 150$ GeV, and $\tan\beta = 2$. $m_{1/2}$ is the gluino mass at the unification scale. The wino and bino masses at the unification scale are given by Eq. (30).

FIG. 6. Same as in the Fig. 5 except that the first generation squark masses are presented.

FIG. 7. Same as in the Fig. 5 except that the Higgs masses are presented.

FIG. 8. Same as in the Fig. 5 except that the chargino and neutralino masses are presented. The horizontal solid line represents the lightest neutralino.

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